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STATISTICAL, NONLINEAR, AND SOFT MATTER PHYSICS

Collapse and Backward Motion of Axisymmetric Toroidal Vortices in an Accretion Flow

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Abstract—The problem of the interaction of two coaxial, counter-rotating ring vortices in the presence of a convergent (accretion) flow with a sink at the center of symmetry has been solved. The vortices that would recede from each other in the absence of a flow (the problem inverse to the Helmholtz problem) are shown to be brought closer together by the flow and then ejected with acceleration along the axis of symmetry. The ejection velocity increases with sink strength. However, if the sink strength exceeds some critical value that depends on the initial conditions, then no ejection occurs and the vortices are captured by the flow and collapse. A similar capture and collapse are also possible during the motion of a single vortex in a flow. The difference from the planar case, where no collapse occurs, is significant. The detected phenomenon can be applied when studying nonlinear processes in atmospheric vortices as well as in active galactic nuclei and planetary atmospheres.

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1. INTRODUCTION

The interaction of ring vortices with flows is of considerable interest and has numerous applications (see the jubilee issue of the journal [1] devoted to the 150th anniversary of Helmholtz's classical work [2] and references therein). In two-dimensional hydrodynamics, a ring vortex is known to be simulated by a pair of point vortices with opposite signs of circulation located symmetrically relative to the axis of motion. In the absence of a flow, the pair moves uniformly and rectilinearly along the axis of symmetry [3]. When a radial flow is taken into account, the character of motion becomes distinctly different [4]. In a divergent flow the distance between the pair components increases and the velocity slows down, while in a convergent flow the situation is reversed: a decrease in the distance between the pair components causes their velocity to increase. In a certain domain of parameters, the vortex can execute a backward motion.

A system of two mirror-symmetric ring vortices (a dipole toroidal vortex) is also of considerable interest in connection with astrophysical applications [5–10]. The dynamics of a dipole toroidal vortex in a radial flow was investigated in a 2D description in [11]. In this approximation, a dipole toroidal vortex can be represented by four point vortices. Starting from the paper by Grobli [12], it is well known that the components of the pairs switch places as a result of their head-on collision, while the new pairs fly apart at a right angle to the direction of motion with the original (in magnitude) velocities. The presence of a central radial flow leads either to deceleration (divergent flow) or acceleration (convergent flow) of the pairs flying apart in opposite directions [11]. The velocity of the pair being ejected in a convergent (accretion) flow depends on the ratio of the flow strength to the vortex intensity and can reach large values. It was shown in [5] that the effect of accelerated vortex pair ejection in a sense is also retained for a more complex accretion flow: the product of the velocities of the ejected pairs is determined only by the monopole component of the flow.

These solutions can be interpreted as the motion and decay of a dipole toroidal vortex into components. From this viewpoint, it was shown in [7, 11] in a 2D description that a dipole toroidal vortex in an accretion flow is compressed along the major radius and decays to produce a two-sided jet of its constituent ring vortices along the system's axis of symmetry.

Here, we consider this problem based on an axisymmetric 3D description of ring (toroidal) vortices (see the reviews [13–17]). The problem admits a Hamiltonian formulation and has an exact solution within the concept of thin toroidal vortices [2]. Significantly, the behavior of 3D vortices differs considerably from the behavior of their analogs in the plane 2D model in a certain domain of parameters. More specifically, collapse with the drawing of vortices into the sink region arises in an intense convergent flow. We show that the backward motion of vortices between the